THE CONSUMER CHOICE CHALLENGE

Polina Rudovska, Olena Radzievska

National University of Food Technologies, Kyiv, Ukraine

April 15–16, 2021, Kyiv

Introduction

- In our report, we will consider the task of consumer choice.
- The task of consumer choice is to choose such a consumer set $\mathbf{x} = \{x_1, x_2, ..., x_k\}$ which maximizes its utility function under a given budget restriction .
- Formally, the task of consumer choice has the form:

$$U(x_1, x_2, ..., x_k) \rightarrow \max$$

$$p_1x_1 + p_2x_2 + \ldots p_kx_k \leq \Im$$

where $U(x_1, x_2, ..., x_k)$ is utility function, $x_1 \geq 0, x_2 \geq 0, \ldots, x_k \geq 0$ and $\Im$ is a budget restriction

- A set of goods $x_1^0, x_2^0, ..., x_k^0$ when the utility function takes the maximum value is called the optimal or local market equilibrium for a consumer .

Consider the problem of choice for Stone model with three benefits

We denote by

$$x_1, x_2, , x_3$$

the consumer set of three goods;
through p_1, p_2, p_3 market prices of one unit of the corresponding good;
through $\mathfrak{I}$- income of an individual, which he is willing to spend on the acquisition of these benefits;
through a_1, a_2, a_3 - the minimum amount of the corresponding good, which is acquired in any case and is not a subject of choice.

In the introduced notation, the task of consumer choice has the form

$$U(x_1, x_2, x_3) = (x_1 - a_1)^{\alpha_1}(x_2 - a_2)^{\alpha_2}(x_3 - a_3)^{\alpha_3} \to \max$$

$$p_1x_1 + p_2x_2 + p_3x_3 \leq \mathfrak{I}$$

For solving this problem we used the method of Lagrange multipliers.

Solution of Problem

Let's write Lagrange function for this problem and examine it on unconditional extremum.

$$L(x_1, x_2, x_3) = (x_1 - a_1)^{\alpha_1}(x_2 - a_2)^{\alpha_2}(x_3 - a_3)^{\alpha_3} + \lambda(p_1x_1 + p_2x_2 + p_3x_3 - \Im)$$

Now we will find partial derivatives of Lagrange functions

$$\frac{\partial L}{\partial x_1} = \alpha_1(x_1 - a_1)^{\alpha_1 - 1}(x_2 - a_2)^{\alpha_2}(x_3 - a_3)^{\alpha_3} + \lambda p_1$$

$$\frac{\partial L}{\partial x_2} = \alpha_2(x_1 - a_1)^{\alpha_1}(x_2 - a_2)^{\alpha_2 - 1}(x_3 - a_3)^{\alpha_3} + \lambda p_1$$

$$\frac{\partial L}{\partial x_3} = \alpha_3(x_1 - a_1)^{\alpha_1}(x_2 - a_2)^{\alpha_2}(x_3 - a_3)^{\alpha_3 - 1} + \lambda p_1$$

$$\frac{\partial L}{\partial \lambda} = p_1x_1 + p_2x_2 + p_3x_3 - \Im$$

The necessary condition of extremum is the equality zero of partial derivatives of Lagrange functions. So we have

$$\frac{\alpha_1 U(x_1, x_2, x_3)}{x_1 - a_1} + \lambda p_1 = 0$$

$$\frac{\alpha_2 U(x_1, x_2, x_3)}{x_2 - a_2} + \lambda p_2 = 0$$

$$\frac{\alpha_3 U(x_1, x_2, x_3)}{x_3 - a_3} + \lambda p_3 = 0$$

$$(p_1 x_1 + p_2 x_2 + p_3 x_3) - \Im = 0$$

Next, we will find expressions for each x_1, x_2, x_3 from these first three equations.

We have such expressions

$$x_1 = a_1 - \frac{\alpha_1 U(x_1, x_2, x_3)}{\lambda p_1} \tag{1}$$

$$x_2 = a_2 - \frac{\alpha_2 U(x_1, x_2, x_3)}{\lambda p_2} \tag{2}$$

$$x_3 = a_3 - \frac{\alpha_3 U(x_1, x_2, x_3)}{\lambda p_3} \tag{3}$$

$$(p_1 x_1 + p_2 x_2 + p_3 x_3) - \mathfrak{I} = 0 \tag{4}$$

We will multiply each of the three first expressions by $\lambda p_i, i = 1, 2, 3$ respectively and then add all these three equalities.

So we have

$$\sum_{i=1}^{3} \alpha_i U(x_1, x_2, x_3) + \lambda \sum_{i=1}^{3} p_i x_i - \lambda \sum_{i=1}^{3} p_i a_i = 0$$

Then using equation (4) we obtain

$$U(x_1, x_2, x_3) \sum_{i=1}^{3} \alpha_i = \lambda(\sum_{i=1}^{3} p_i a_i - \mathfrak{I})$$

Hence

$$\frac{U(x_1, x_2, x_3)}{\lambda} = \frac{\sum_{i=1}^{3} p_i a_i - \mathfrak{I}}{\sum_{i=1}^{3} \alpha_i}$$

Next we substitute the right hand side of this equality into equalities 1-3 and we have such results

Conclusion

$$x_1 = a_1 + \frac{\alpha_1(\mathfrak{I} - \sum_{i=1}^{3} p_i a_i)}{p_1 \sum_{i=1}^{3} \alpha_i}$$

$$x_2 = a_2 + \frac{\alpha_2(\mathfrak{I} - \sum_{i=1}^{3} p_i a_i)}{p_2 \sum_{i=1}^{3} \alpha_i}$$

$$x_3 = a_3 + \frac{\alpha_3(\mathfrak{I} - \sum_{i=1}^{3} p_i a_i)}{p_3 \sum_{i=1}^{3} \alpha_i}$$

The obtained representation for the demand function has the following economic interpretation. First, the minimum required amount of each good is acquired. Then the amount of money remaining after this is calculated, which is distributed in proportion to the weights α_i . Dividing the amount of money by the price p_i , we get an additional amount of the i-th good purchased over the minimum.